Reper-Projective Architecture of Formula Chains: PILOT-01

Final bilingual preprint review with the Fano Plane as an Ontological Barrier

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Abstract

This preprint presents PILOT-01, a narrow-domain test of the Reper-Projective Database (RPD) program. The article models mathematical texts as graphs of formula-chain transitions, Reper nodes, gap nodes, obstruction carriers, and theorem candidates. The selected domain combines projective-harmonic geometry, packet incidence, corrected obstruction notation, and Fano-type projective carriers. The revision verifies the external reference boundary: cross-ratio, projectivization, projective spaces, and the Fano plane are treated as classical background, while the RPD contribution consists of formula-chain auditing, the Formula-Chain Obstruction Complex (FCOC), a nontrivial packet-obstruction example, and a conditional Fano carrier theorem. The Fano plane is interpreted as an ontological barrier: local packet-obstruction carriers do not automatically globalize into $\mathbb{P}^2(\mathbb{F}_2)$; crossing the barrier requires explicit point-compatible identification maps.

Keywords: Reper-Projective Database; formula-chain audit; projective geometry; cross-ratio; Fano plane; packet incidence; obstruction carrier; FCOC; theorem-candidate detection.

Contents

1 Introduction

Mathematical texts contain explicit formulas and proofs, but also hidden domain assumptions, under-specified transitions, local gaps, and possible theorem candidates. The Reper-Projective Database (RPD) program models such objects as a graph. PILOT-01 is the first narrow-domain article in this program. It focuses on projective-harmonic formula chains, packet incidence, corrected obstruction notation, and Fano-type carriers.

The guiding editorial rule is

$$\boxed{\text{classical known fact} \neq \text{RPD method theorem} \neq \text{internal construction theorem} \neq \text{conditional th}}$$

This rule is essential because PILOT-01 uses classical objects such as cross-ratio, projectivization and the Fano plane, while the RPD contribution lies in how these objects are organized into an audit and obstruction architecture.

2 Related Work and Prior-Art Boundary

The cross-ratio and the harmonic value -1 are classical objects of projective geometry. PILOT-01 does not claim novelty for the cross-ratio. The RPD contribution is the use of a Reper quadruple $(R, I, U; D)$ and the corresponding formula-chain audit rule that prevents a harmonic condition from being treated as a fully justified truth-status without an admissible domain and a sufficient foundation.

Projectivization is also standard. A projective object is not the same as the vector space from which it is formed. This motivates the corrected obstruction notation

$$O_{\mathrm{OB}} \quad \text{vector obstruction carrier}, \qquad \mathbb{P}(O_{\mathrm{OB}}) \quad \text{projectivized obstruction carrier}.$$

The Fano plane is a classical finite projective plane over $\mathbb{F}_2$. PILOT-01 uses it conditionally: a three-dimensional $\mathbb{F}_2$ obstruction carrier projectivizes to $\mathbb{P}^2(\mathbb{F}_2)$, and a global Fano carrier requires an explicit identification axiom.

Maurer-Cartan and deformation theory provide an external analogy for quadratic obstruction terms and lifting problems. This article does not claim that the internal FCOC obstruction carrier is already identical to a classical deformation-theoretic obstruction quotient. That comparison remains open.

3 Formal Definitions

Definition 3.1 (Formula-chain step)**.** A formula-chain step is a typed transition

$$s = (F_i, F_j, \tau, A, \mathrm{Dom}, D),$$

where F_i and F_j are formulas or formula nodes, τ is the transition type, A is a support set, Dom is an admissible domain, and D is a sufficient foundation.

Definition 3.2 (Reper quadruple)**.** A Reper quadruple is written

$$\mathrm{Rep} = (R, I, U; D),$$

where R denotes established content, I an invariant or idea, U a possibility field, and D a sufficient foundation or context.

Definition 3.3 (Formula-Chain Obstruction Complex)**.** A finite-dimensional Formula-Chain Obstruction Complex is a three-term complex

$$C^0 \xrightarrow{d_0} C^1 \xrightarrow{d_1} C^2$$

with a quadratic obstruction map $q_{\mathrm{OB}} : C^1 \to C^2$. Define

$$H_{\mathrm{OB}} = C^2 / \operatorname{im}(d_1), \qquad O_{\mathrm{OB}} = \operatorname{span}(\operatorname{im}(\pi \circ q_{\mathrm{OB}})) \subseteq H_{\mathrm{OB}}.$$

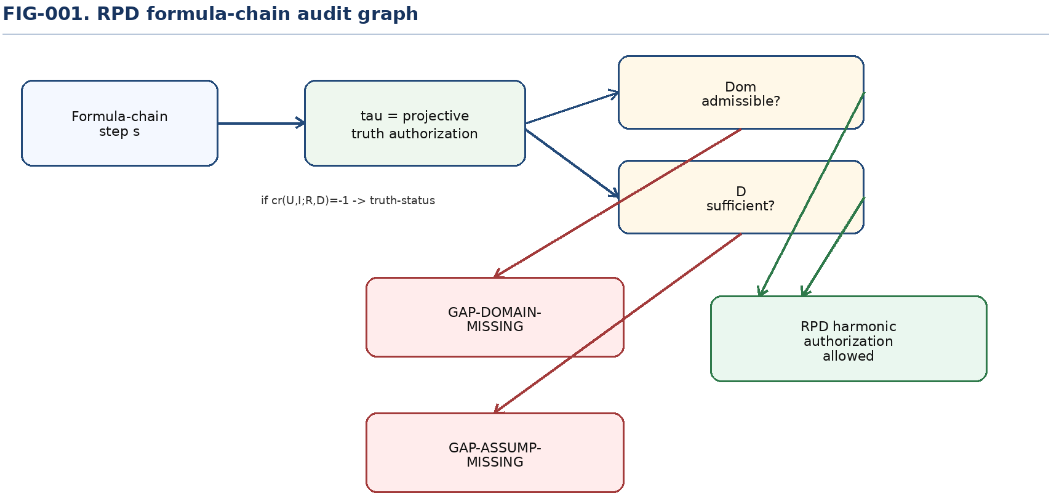

Figure 1: RPD formula-chain audit graph.

4 Formula-Chain Gap Theorem

Theorem 4.1 (Formula-chain gap theorem)**.** *If a formula-chain step uses projective truth authorization*

$$\operatorname{cr}(U, I; R, D) = -1 \implies \textit{truth-status}$$

without an admissible domain or a sufficient foundation D, then the RPD audit creates `GAP-DOMAIN-MISSING` or `GAP-ASSUMP-MISSING`.

Proof. The RPD audit contains two rules. `AUDIT-DOMAIN` creates `GAP-DOMAIN-MISSING` when the domain of a projective truth authorization step is absent or inadmissible. `AUDIT-FOUNDATION` creates `GAP-ASSUMP-MISSING` when D is absent, empty, ambiguous, or not attached to the step. Therefore, if either requirement is missing, at least one of the two gap nodes is generated. □

5 Corrected Obstruction Notation and Projectivization

Theorem 5.1 (Projectivization dimension lemma)**.** *Let O be a nonzero vector space over a field k with $\dim_k O = n + 1$. Then $\mathbb{P}(O)$ is isomorphic to $\mathbb{P}^n(k)$ after choosing a basis.*

Proof. A basis identifies O with k^{n+1}. Projectivizing gives $\mathbb{P}(O) \cong \mathbb{P}(k^{n+1}) = \mathbb{P}^n(k)$. ☐

Corollary 5.2 (Fano and real projective regimes)**.** *If* $\dim_{\mathbb{F}_2} O_{\mathrm{OB}} = 3$*, then* $\mathbb{P}(O_{\mathrm{OB}}) = \mathbb{P}^2(\mathbb{F}_2)$*. If* $\dim_{\mathbb{R}} O_{\mathrm{OB}} = 3$*, then* $\mathbb{P}(O_{\mathrm{OB}}) = \mathbb{RP}^2$*.*

6 FCOC Carrier Theorem

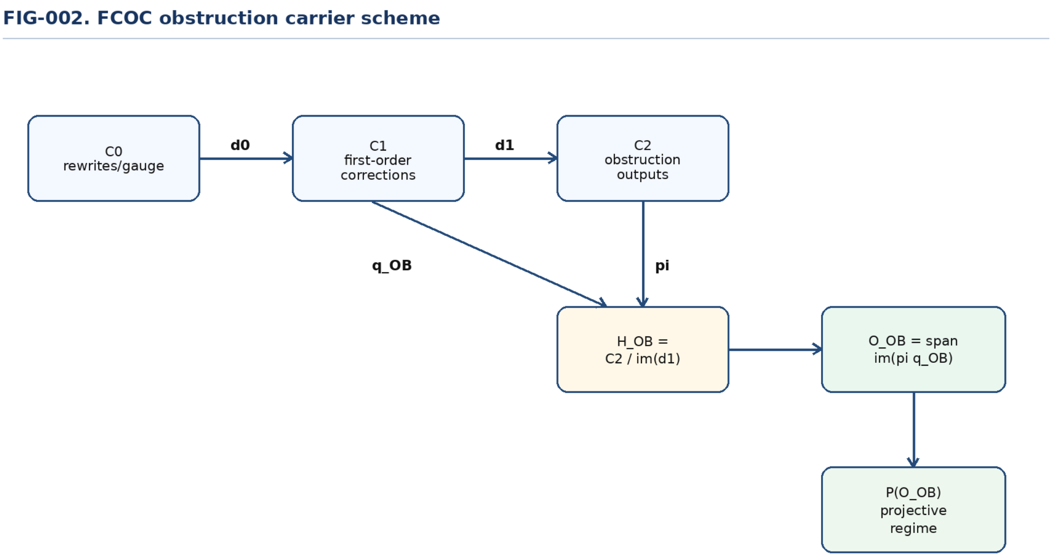

Figure 2: FCOC obstruction carrier scheme.

Theorem 6.1 (FCOC carrier theorem)**.** *Given a finite-dimensional internal complex* $C^0 \to C^1 \to C^2$ *and* $q_{\mathrm{OB}} : C^1 \to C^2$*, the quotient* $H_{\mathrm{OB}} = C^2/\operatorname{im}(d_1)$ *is a vector space and* $O_{\mathrm{OB}} = \operatorname{span}(\operatorname{im}(\pi \circ q_{\mathrm{OB}}))$ *is a vector subspace. Thus* O_{OB} *is a well-defined vector obstruction carrier.*

Proof. Since $\operatorname{im}(d_1)$ is a subspace of C^2, the quotient H_{OB} is a vector space. The image of $\pi \circ q_{\mathrm{OB}}$ is a subset of H_{OB}. The span of a subset of a vector space is a vector subspace. Hence O_{OB} is a vector subspace of H_{OB}. ☐

7 Nontrivial Packet Obstruction

Theorem 7.1 (Nontrivial packet obstruction)**.** *Let* $C^1 = k^3$*,* $C^2 = k^3$*,* $d_1 = 0$ *and*

$$q_{\mathrm{OB}}(a, b, c) = (ab, bc, ca).$$

Then $O_{\mathrm{OB}} = k^3$*.*

Proof. Since $d_1 = 0$, we have $H_{\mathrm{OB}} = k^3$. Evaluating q_{OB} gives

$$q_{\mathrm{OB}}(1, 1, 0) = e_1, \qquad q_{\mathrm{OB}}(0, 1, 1) = e_2, \qquad q_{\mathrm{OB}}(1, 0, 1) = e_3.$$

The image of q_{OB} contains a basis of k^3. Therefore its span is k^3, so $O_{\mathrm{OB}} = k^3$. ☐

FIG-003. Nontrivial packet obstruction map

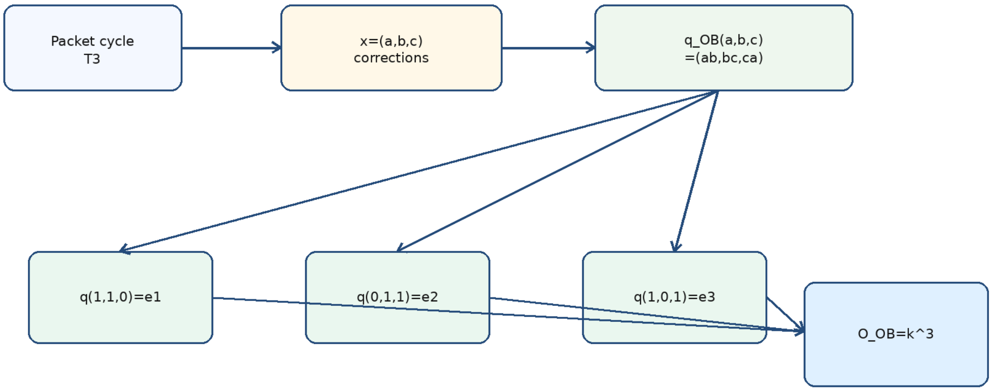

Figure 3: Nontrivial packet obstruction map.

8 Fano Plane as Ontological Barrier

The Fano plane is not merely a finite projective example in PILOT-01. It is an ontological barrier between local packet-obstruction carriers and a global projective carrier. Before the barrier, seven local carriers produce a direct sum of dimension 21. After the barrier, point-compatible identification maps $\phi_L : O_L \to \mathbb{F}_2^3$ allow the identified carrier $O^{\mathrm{id}}_{\mathrm{global}}$ to be isomorphic to $\mathbb{F}_2^3$, and $\mathbb{P}(O^{\mathrm{id}}_{\mathrm{global}}) = \mathbb{P}^2(\mathbb{F}_2)$.

Principle 8.1 (Fano ontological barrier)**.** *The local carriers* $\{O_L\}_{L\in\mathrm{FanoLines}}$ *do not automatically determine a global Fano carrier. The honest pre-barrier globalization is the direct sum of seven local three-dimensional carriers, hence dimension* 21*. Crossing the barrier requires explicit point-compatible maps* $\phi_L : O_L \to \mathbb{F}_2^3$*.*

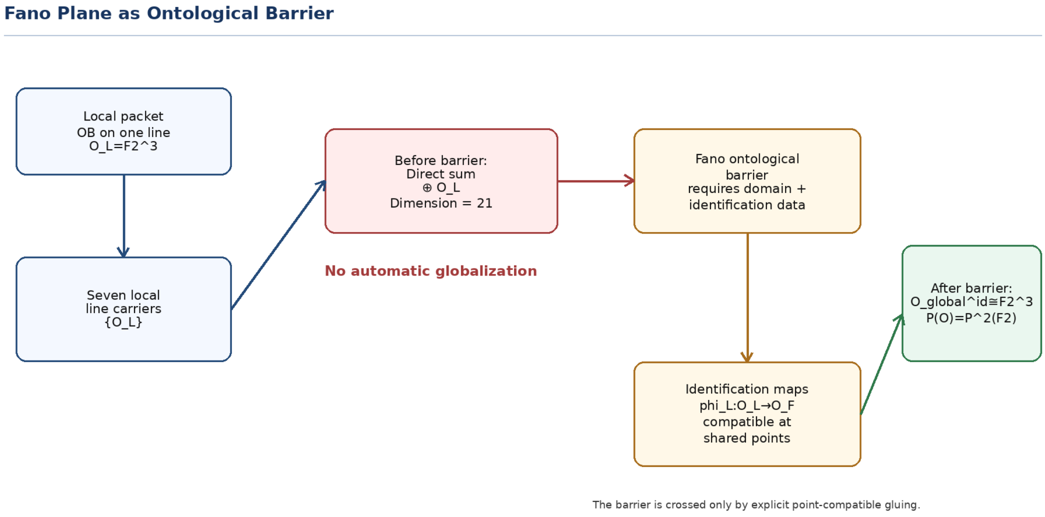

Figure 4: Fano plane as an ontological barrier between local packet-obstruction carriers and a global projective carrier.

9 Conditional Fano Carrier Theorem

Theorem 9.1 (Conditional Fano carrier)**.** *Under the Fano identification axiom, local line carriers are glued by point-compatible maps. The quotient/colimit is generated by the seven nonzero vectors of* $\mathbb{F}_2^3$*. Hence*

$$O_{\text{global}}^{\text{id}} \cong \mathbb{F}_2^3, \qquad \mathbb{P}(O_{\text{global}}^{\text{id}}) = \mathbb{P}^2(\mathbb{F}_2).$$

Proof. The compatibility axiom identifies all local copies of the same Fano point direction. After quotienting by these relations, the global carrier is generated by the seven nonzero vectors of $\mathbb{F}_2^3$. These vectors generate $\mathbb{F}_2^3$. Projectivizing gives $\mathbb{P}^2(\mathbb{F}_2)$. □

10 Claim Boundary and Open Problems

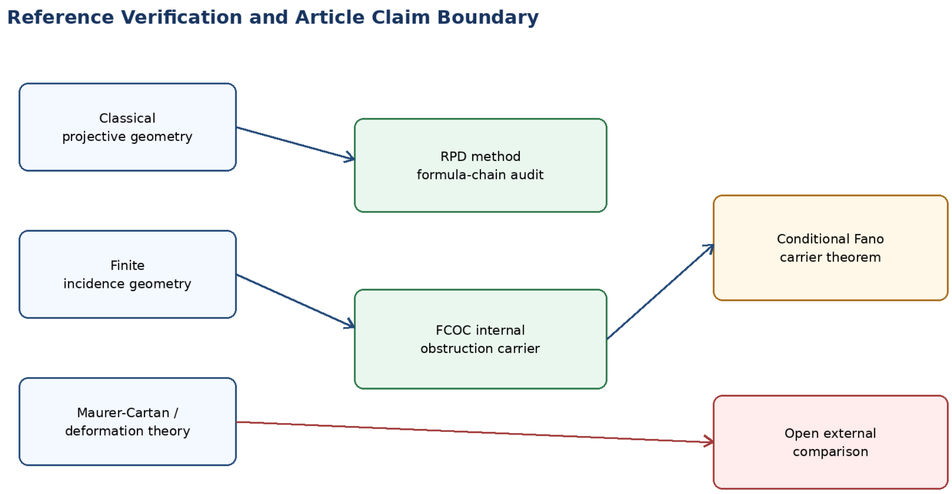

Figure 5: Verified claim boundary: classical background, internal RPD results, conditional theorem, and open candidates.

The main open problems are: a general packet incidence reflection theorem; a natural packet functor producing the Fano identification maps; a rigorous comparison with Maurer-Cartan/deformation-theoretic obstruction quotients; and a proof-assistant or executable checker implementation of the RPD audit rules.

11 Conclusion

The LaTeX preprint pack consolidates PILOT-01 as a publication-oriented mathematical artifact. Its strongest results remain internal or conditional, while external deformation-theoretic claims are explicitly left open. The Fano plane is no longer a decorative example: it is the ontological barrier that separates local packet-obstruction data from a global projective carrier.

A Theorem and Status Table

ID	Name	Status
T-PH-009	Reper harmonic method lemma	method lemma, not classical novelty
T-PH-001	Formula-chain gap theorem	proved inside RPD method
T-PH-002	Projectivization dimension lemma	classical known fact used
T-PH-003	Fano/RP2 corollaries	conditional on field/dimension

ID	Name	Status
T-PH-004	FCOC carrier theorem	proved inside internal RPD model
T-PH-005	Nontrivial packet OB theorem	proved inside internal RPD model
T-PH-006	Fano local line extension	proved inside internal RPD model
T-PH-007	Fano direct-sum carrier theorem	proved inside internal RPD model
T-PH-012	Fano ontological barrier principle	RPD interpretation and internal boundary principle
T-PH-008	Conditional Fano carrier theorem	conditional on identification axiom
T-PH-010	General packet incidence reflection theorem	open
T-PH-011	External deformation-theoretic OB theorem	open

B Dependency Graph

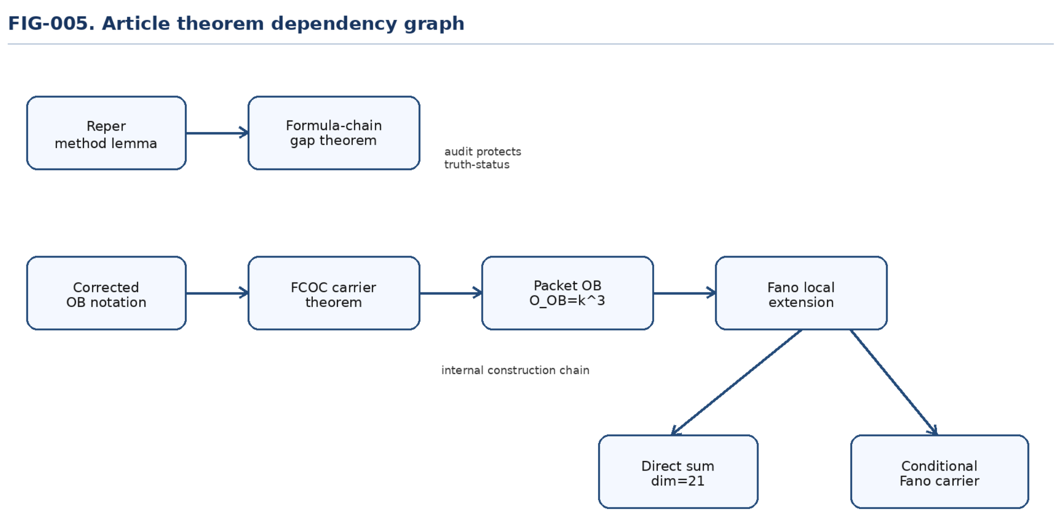

Figure 6: Article theorem dependency graph.

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